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Reduction of Noise by the Use of Damping Materials

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Phil. Trans. R. Soc. Lond. A 1968 **263**, 441-453
doi: 10.1098/rsta.1968.0028

Reduction of noise by the use of damping materials

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Introduction

Vibration damping materials are used to damp out the bending vibrations of sheet-metal structures. In this way, weakly damped natural and resonant vibrations of the sheet-metal which lead to radiation of disturbing noise into the adjacent air are suppressed, the noise level being thus reduced. Free bending and strain waves can transport sound energy via extended sheet-metal structures to points far away from the sources, causing sound radiation into the air. In this case too, the application of vibration damping materials will be beneficial because in this way the structure-borne sound propagation will be partially suppressed by the attenuation of the free-bending waves. Furthermore, the damping of the bending vibrations can essentially improve the sound insulation of light-weight structures, so-called sandwich walls, consisting of outer metal plates and an air cushion between, filled with sound-absorbing material. The damping of the metal plates diminishes the influence of resonances and coincidences which otherwise would lead to a diminution of the sound reduction of the sandwich wall (Stüber 1956, 1965). Finally, it has been shown that the fatigue life of metal panels, for example near the outer walls of jets which are excited to flexural vibrations by the random pressure of the air-borne sound field of high intensity, is increased as a result of the damping of the panels (Kurtze & Westphal 1965).

There has been a systematic development of vibration damping materials for nearly 20 years. Formerly, the production of these materials was a matter of trial-and-error methods (Beranek 1960). It was a long road to the high-quality products which are in use today. The measuring technique had to be developed with which the viscoelastic properties, especially the damping of the damping materials and of the combined systems consisting of metal plates or sheet metal and damping viscoelastic layers, could be investigated. Furthermore, the theory of the bending vibrations and the viscoelastic properties, especially the damping of the combined systems, had to be worked out to enable us to understand the dependence of those properties on the viscoelastic properties of the layers and their thicknesses. In the first stage, two-layer systems consisting of metal panels with a damping layer on one side were considered (Lienard 1951, 1957; Oberst & Frankenfeld 1952; Oberst, Becker & Frankenfeld 1954). From the results of the theory the conditions which the damping materials had to fulfil to provide high damping of the combined systems could be established. It soon became obvious that the highest damping values could be obtained with amorphous thermoplastic polymers. Next, the possibilities were studied as to how vibration damping materials with optimum damping efficiency could be obtained in prescribed ranges of the frequency and temperature. Thus, after high polymer physics, high polymer chemistry came into play.

At a later stage, theories of multiple-layer-systems, especially sandwich systems in another sense consisting of outer metal layers and a viscoelastic damping layer between, were worked out in the same way (Kerwin 1959; Ross, Ungar & Kerwin 1959; Kurtze & Watters 1959; Naumkin, Tartakowskij & Effussi 1959; Ungar & Ross 1959; Tartakowskij & Rybak 1962; Yu & Ren 1965). In this case, too, thermoplastic materials could be developed which produced optimum damping of the combined systems in prescribed ranges of frequency and temperature (Oberst, Bohn & Linhardt 1961; Oberst & Schommer 1965; Oberst 1966, 1967).

Vibration damping thermoplastic materials with optimum efficiency in prescribed ranges of frequency and temperature

The viscoelastic properties of the vibration damping materials are usually described in terms of the complex Young modulus E^*, which is measured in the case of stationary excitation of forced vibrations of bar-shaped specimens by the circular frequency $\omega = 2\pi f$, where $f =$ the frequency, $E^* = E' + jE''$, and $j = \surd(-1)$. E', the dynamic modulus of elasticity or 'storage modulus', is a measure of the dynamic stiffness, E'', the so-called 'loss modulus', is a measure of the internal energy losses. Besides the loss modulus, the 'loss factor' $d = E''/E'$ is chosen as a relative measure of the energy losses; $d = \tan\delta$ where δ is the phase angle between the stress and the strain.

It is usual to determine the viscoelastic values by investigating the bending vibrations of bars or strips of the material under test, i.e. the weakly damped free vibrations or the resonant vibrations in the case of stationary excitation, making use of free-free or free-clamped bars (in the terminology of Lord Rayleigh) (Oberst & Frankenfeld 1952; Oberst *et al.* 1961; Becker 1955; Oberst 1961). The viscoelastic properties of combined systems, consisting of sheet-metal and viscoelastic damping layers, can be measured by the same method.

The highest internal damping is obtained with amorphous high polymers in a certain temperature range. Figure 1 shows schematically curves of the viscoelastic values E', E'' and d against temperature as they are obtained if the measuring frequency is kept constant. Curves of this type illustrate the properties of the polymer materials in the different ranges of the temperature. At low temperatures the material is in the glassy state and E' has high values in the order of magnitude of 4×10^4 kgf/cm^2, whereas E'' and d have relatively low values. Above the so-called glass transition temperature T_g the long molecular chains of the high polymers become mobile, segments of them may change their mutual positions, and in the case of mechanical stresses relaxation processes of the segments occur, leading to a phase shift between the stress and the strain and consequently to hysteresis losses (Staverman & Schwarzl 1956; Tobolsky 1960; Ferry 1961; Nielsen 1962; Becker & Schreuer 1962). In this temperature range the viscoelastic material softens. The values of the storage modulus E' decrease over several powers of 10, and the loss modulus and the loss factor pass through high maxima. In the case of homopolymers, the long molecular chains consist of monomer components of equal shape. The transition range in which the material softens has a width of *ca.* 50 °C. The maximum of the loss factor has values of the order of 1 in this range. Such high values cannot be obtained with any other material, and this is the reason why amorphous high polymers are most suitable as vibration damping materials. Above the transition range, cross-linked amorphous polymers are in the well-known rubbery state. Non-cross-linked thermoplastic polymers are in a quasi-rubbery state where the rubber-like behaviour is superimposed by a certain flow which increases with rising temperature.

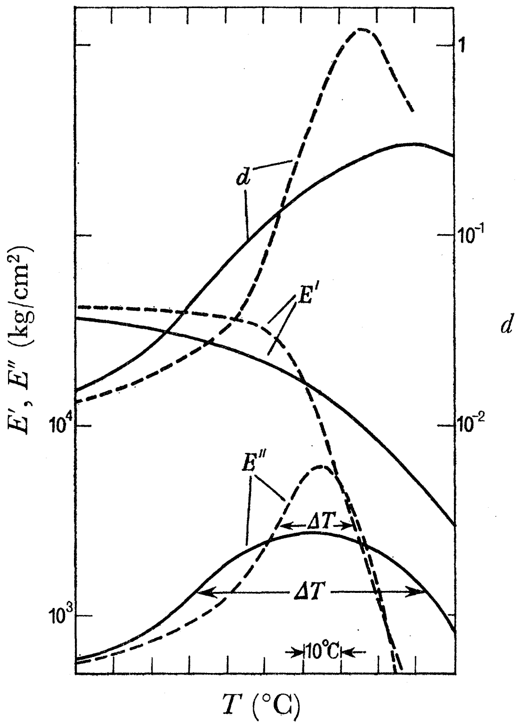

Figure 1. Temperature curves of the dynamic modulus of elasticity E', the loss modulus E'' and the loss factor d at a given frequency for a hypothetical vibration damping material; ΔT the temperature bandwidth (half-width).

With increasing frequency the temperature curves (figure 1) are shifted in the direction of higher temperatures. In the case of homopolymers, the relations between the temperature and the frequency dependence are well known. They are governed by the so-called WLF-function (Staverman & Schwarzl 1956; Tobolsky 1960; Ferry 1961; Nielsen 1962; Becker & Schreuer 1962). Furthermore, the 'relaxation spectrum' of the molecular relaxation processes, i.e. the distribution of relaxation times, in the transition range is well known, too. It has been calculated by making use of molecular chain statistics (Rouse 1953). The quantity which is most suitable for the characterization of the damping efficiency of a vibration damping material is the loss modulus E''. From the laws which govern the molecular behaviour of the amorphous homopolymers we know that the maximum in the transition range has a half-width $\Delta T = 20$ °C (see figure 1). This width is not sufficient for many technical applications of the vibration damping materials. For instance, in vehicles for which the maximum of E'' is situated at 20 °C, the damping efficiency at 0 °C would be too low. Therefore, it is necessary to increase the half-width ΔT. It is possible to obtain vibration damping materials with optimum damping efficiency in prescribed ranges of the frequency and in ranges of the temperature with prescribed position and width by mixing suitably chosen compatible amorphous polymers with slightly different positions of the glass temperatures T_g, or by copolymerization of suitably chosen monomers. Furthermore, plastification, filling of the materials and other measures, can help to obtain the desired optimum efficiency (Oberst *et al.* 1961; Oberst & Schommer 1965; Oberst 1966).

In this way temperature curves like those with a larger half-width ΔT of E'' as shown in figure 1 can be obtained. Assuming that the physical laws which are valid for homopolymers can also be used as an approximation for the mixtures, and copolymers with increased half-width there can be derived a relation between the half-width ΔT and the maximum value $E''_{max.}$ of the loss modulus:

$$\Delta T E''_{max.}/E_g \approx \text{const.}$$

E_g is the glass modulus, i.e. the value of E' in the glassy state or at high frequencies, respectively. The equation shows that the increase of the half-width leads to a corresponding decrease of the maximum damping. This law has been called the temperature bandwidth law (Oberst *et al.* 1961; Oberst 1966, 1967; Linhardt & Oberst 1961). As figure 1 shows, the increase in the temperature bandwidth at the same time leads to a flattening of the temperature curve of E'. This relation is important with respect to the limits of the damping efficiency which cannot be exceeded in principle. It is useful from a technical viewpoint to know these limits.

Optimized combined systems of sheet metal and thermoplastic damping layers

As already mentioned, we have to distinguish between two-layer systems and various types of multiple-layer panels. Figure 2 shows schematically the systems which have so far been of interest. Besides the two-layer systems consisting of sheet metal with a damping layer on one side there are in practical use three-layer sandwich panels, consisting of two outer panels and the damping layer between. Symmetrical and unsymmetrical systems are being used in which the sheet metal layers have different thicknesses, and furthermore panels with thin and thick damping layers. The thin layers are especially of interest in the case of self-adhesive damping materials which at the same time are active as adhesives and as vibration damping materials. The thick damping layers are mostly thermoplastic films bonded to the metal panels.

(a) *Two-layer systems*

The results of the theory of the bending vibrations of sheet-metal with a damping layer on one side have proved themselves to be of importance for the development and the manufacture of such systems. The example of figure 3 may illustrate these relations (Oberst *et al.* 1961; Oberst 1966, 1967). In this figure, the values E'', E' and d of an optimized thermoplastic material are shown in dependence on the temperature for 200 Hz. Furthermore, there is included the loss factor of the two-layer system consisting of sheet-metal and a sprayed-on layer of the damping material on it, the mass ratio of the layers amounting to 20%. This is a ratio which is often used for comparison purposes. The combined system is in this case considered to be a homogeneous system, on which free bending waves are possible like those on panels of a homogeneous material. The loss factor $d_{comb.}$ can immediately be compared with the loss factors of pure viscoelastic materials.

The damping material is a modified vinyl acetate copolymer filled with Vermiculite, an expanded mica-like material. For comparison, the viscoelastic values of the non-filled material are contained in the figure. Figure 3 shows that the maximum of the loss modulus of the non-filled material is situated for 200 Hz at 10 °C. The material has a moderate temperature bandwidth. The stiffness of the damping material, i.e. the modulus E', is already somewhat increased in the glassy state by the use of Vermiculite as filling material.

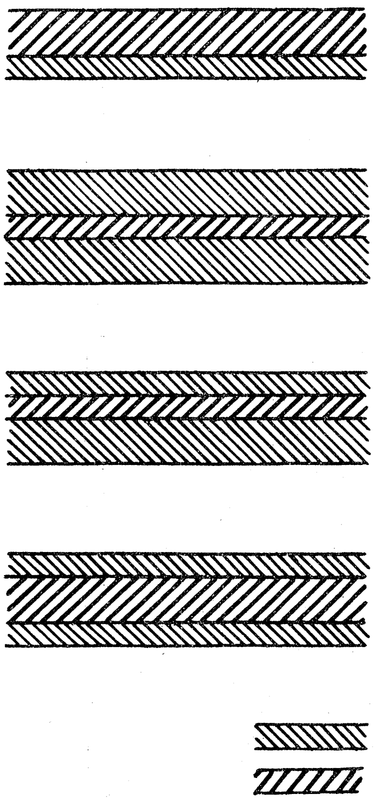

FIGURE 2. Composite systems of metal plates and viscoeleastic damping layers.

In the transition range the stiffening efficiency of the filler is considerable. At the same time, also the loss modulus E'' of the filled material is higher in this temperature range, which results in a considerable broadening of the temperature bandwidth. The theory of the two-layer systems shows that at reasonable thickness or mass ratios, the loss factor $d_{\text{comb.}}$ of the combined system is approximately proportional to E''. Figure 3 confirms that this is in fact the case. The viscoelastic layer containing Vermiculite has a relatively low density. This is a favourable condition in the present case, because according to the theory the loss factor $d_{\text{comb.}}$ of the combined system is approximately proportional to the square of the thickness ratios in the range of thickness ratios which are of technical interest. If the mass ratio (20 %) is prescribed larger thicknesses of the damping layer can be applied with materials of low density, so that relatively high values $d_{\text{comb.}}$ can be obtained.

It is seen from figure 3 that the present two-layer system has been optimized for a temperature range from 0 to 50 °C, the maximum of damping of the combined system being situated at 20 °C. For the definition of the temperature bandwidth a reference level $d_{\text{comb.}} = 0{\cdot}05$ has been chosen. This level is at least 10 dB above the damping values of sheet-metal panels in metal structures to which no damping material has been applied. The damping of such systems corresponds to values $d_{comb.} \leqslant 0{\cdot}01$ (Oberst *et al.* 1961; Oberst 1966, 1967; Heckl 1962, 1963). The present system has been optimized for application for instance in vehicles. According to the temperature bandwidth law the maximum value $d_{comb.}$ is of the order of 0·1. With a corresponding homopolymer material a higher maximum can be obtained, but the temperature bandwidth would be too small.

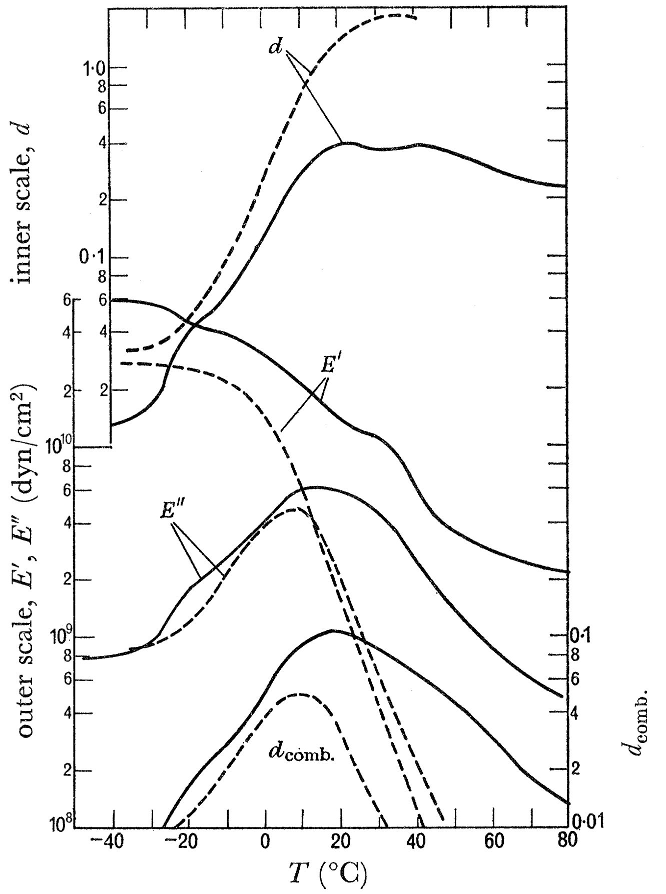

FIGURE 3. Values E', E'' and d of a damping material and loss factor $d_{comb.}$ of sheet steel with a damping layer in dependence on the temperature for 200 Hz. Damping material, a vinyl acetate copolymer. - - - - -, Unfilled; ——, filled with Vermiculite; mass ratio of the layers of the two-layer system 20 %.

All the curves in figure 3 shift in the direction of higher temperatures, as the frequency increases. An increase of the frequency by a factor of 10 corresponds to a shift of 6 to 10 °C (compare the following figures). Thus it is sufficient to describe the viscoelastic properties by a set of curves for one single frequency (200 Hz).

(*b*) *Three-layer sandwich panels*

The theory of multiple layer systems which has been developed in a similar way as the theory of the two-layer systems (Ross *et al.* 1959) results in series of equations, the solutions of which do not lead to explicit expressions for the interesting viscoelastic quantities, especially the damping of the composite structure. Therefore, it is necessary to use an electronic computer. Results obtained in this way have been confirmed experimentally. In the case of the sandwich panels the influence of the 'geometry' of the systems comes much more into play than in the case of the two-layer systems. For instance, the temperature curves of the loss factor $d_{\text{comb.}}$ of the composite structure are shifted with increasing frequency in the direction opposite to that of the shift of the curves of the viscoelastic values of the damping material. Nevertheless, it is also possible in this case to develop optimized systems for special applications. The following examples serve to illustrate these conditions (Oberst *et al.* 1961; Oberst & Schommer 1965; Oberst 1966, 1967).

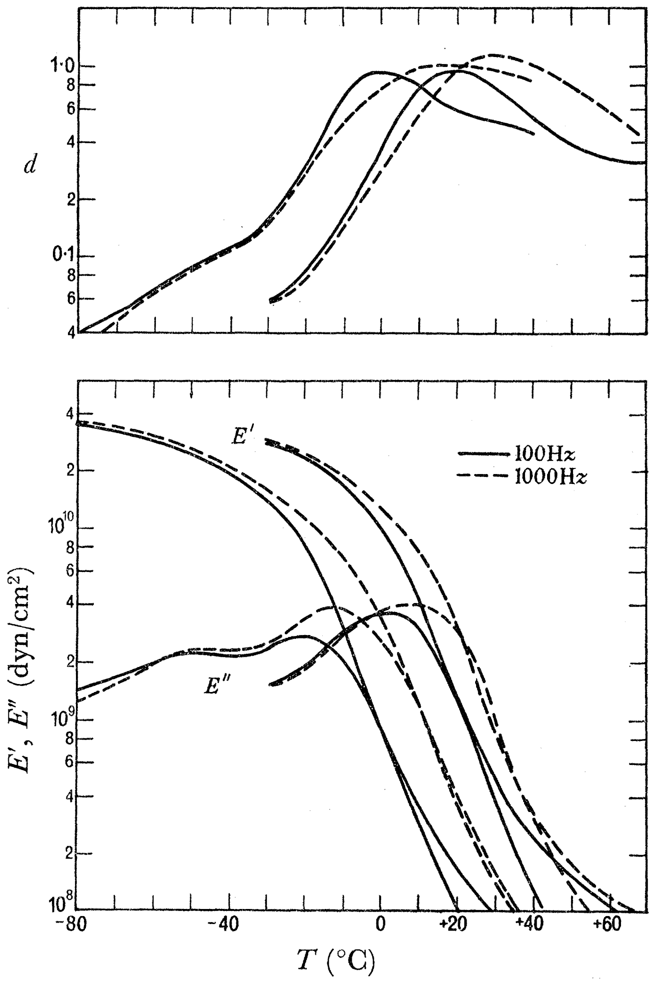

FIGURE 4. Values E', E'' and d of copolymers of vinyl chloride and an acrylate with different weight ratios of the monomers in dependence on the temperature.

Figure 4 shows the viscoelastic values of a vinyl chloride (*VC*) copolymer which has been copolymerized of *VC* and an acrylate, the homopolymer of which has a low position of the glass transition temperature T_g far below 0 °C. The copolymerization can be performed in such a way that the curves of figure 4 are obtained. These curves relate to two different weight ratios of the monomers. It is seen from the figure that temperature broad-band materials are obtained in this way with broad maxima for the loss modulus and the loss factor, and a relatively flat decrease of the modulus E' in the transition range. By changing the weight ratio the range of optimum damping efficiency can be shifted. Furthermore,

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the figure illustrates the shift of the curves when the frequency varies from 100 to 1000 Hz, the frequency range in between is that of the greatest technical interest.

Foils made of these copolymers are bonded to sheet-metals. In this way sandwich panels are formed which can be considered systems with a rather thick damping layer. Figure 5 shows curves of the loss factor $d_{\text{comb.}}$ of such systems, consisting of two outer sheet-metal panels of a thickness of 1 mm and thicknesses of the damping foils varying in a broad range between 0·2 and 4 mm. The series of curves illustrate the strong dependence of the

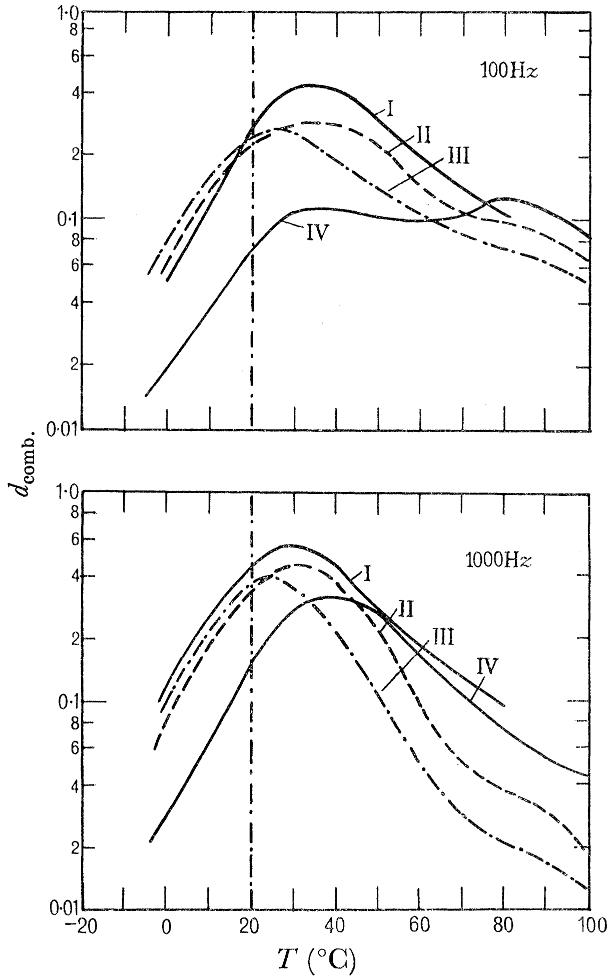

FIGURE 5. Loss factor $d_{\text{comb.}}$ of three-layer sandwich panels with intermediate damping foils of different thicknesses. Damping material one of the vinyl chloride copolymers of figure 4 (pairs of curves at the right side); sheet steel thickness 1 mm, intermediate layer thicknesses 4 mm (curves I), 2 mm (curves II), 1 mm (curves III) and 0·2 mm (curves IV).

system on the geometry, that means, on the thicknesses of the different layers. Furthermore, it is seen from this figure that much higher maximum values of the loss factor $d_{\text{comb.}}$ can be obtained with such systems than with two-layer ones. Figure 5 relates to the pair of curves in figure 4 with a higher temperature position. The damping layer in the vibrating sandwich panel is under shear stress, whereas the damping layer on one side in the two-layer system is under simple uniaxial longitudinal stress. This is the main reason why higher maximum values of the damping of the composite structure are obtained with the sandwich panel. Considerable temperature bandwidths are obtained with these systems, the bandwidth referring to the above mentioned reference level (see figure 5).

The following examples are intended to illustrate the possibilities in the case of thin self-adhesive damping layers. Figure 6 shows the temperature curves of E', E'', and d for such a material, namely, a modified vinyl acetate copolymer which can be applied as an adhesive at elevated temperatures in the melt. The curves have been obtained in the range below 20 °C by making use of the bending vibration method. At higher temperatures they

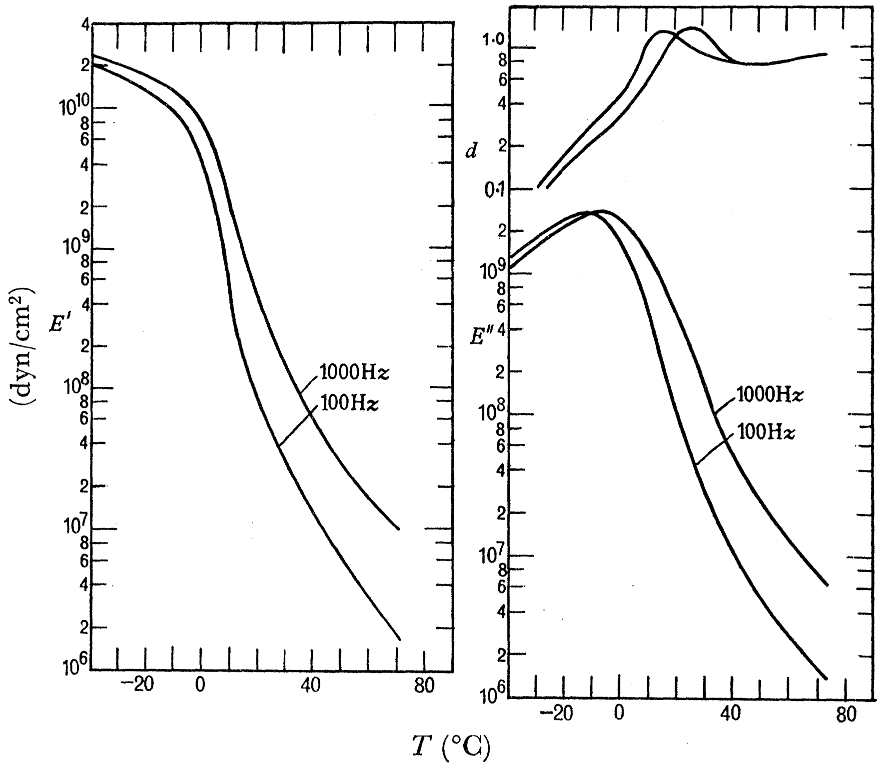

FIGURE 6. Temperature curves of E', E'' and d of a self-adhesive modified vinyl acetate copolymer.

have been determined indirectly by means of the results of the theory and of the measured viscoelastic values of combined sandwich systems with different thicknesses of the sheet-metal panels, making use of the computer (H. Braunisch, unpublished). The material under test is such a one of moderate temperature bandwidth. The loss factor $d_{\text{comb.}}$ of sandwich systems of any thicknesses of the metal panels can be calculated from the visco-elastic values and the thicknesses of the layers, by means of the computer. The calculated values agree with experimental results.

As an example, figure 7 shows a series of temperature curves of the loss factor $d_{\text{comb.}}$ of sandwich systems in which the thicknesses of one panel (0·5 mm) and the damping layer (0·3 mm) have been kept constant and the thickness of the second panel has been varied in a broad range between 0·5 and 10 mm. The highest values of $d_{\text{comb.}}$ are obtained with the symmetrical system, both panels being 0·5 mm thick. The maximum value $d_{\text{comb.}} = 0{\cdot}7$ has nearly the magnitude of the loss factor of the viscoelastic material itself. This means that these maximum values can be scarcely exceeded by any other composite structures. The damping maximum and at the same time the temperature bandwidth decrease with increasing ratio of the thicknesses of the panels. Satisfying curves are obtained up to thickness ratios of the order of 4. Thus, one reads from figure 7 that the asymmetry of the systems should not be too high. Asymmetrical sandwich panels are often necessary with regard to the static stiffness. In a similar way many other questions can be answered, so that one concludes that three-layer systems can also be well controlled.

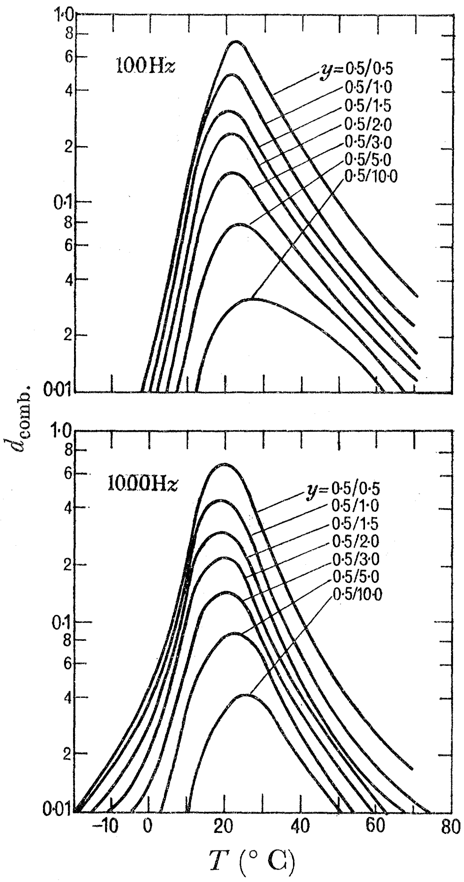

FIGURE 7. Loss factor $d_{comb.}$ of asymmetrical three-layer sandwich panels. Damping layer self-adhesive copolymer of figure 6, thickness 0·3 mm; one steel plate thickness 0·5 mm. y: thickness ratios of the steel plates.

(c) *Optimization of viscoelastic damping materials for specific structural composite applications*

Figure 8 gives a survey on the damping efficiencies of a manifold of multilayer systems, which demonstrates that it is possible in fact to optimize such systems for various technical applications (Oberst & Schommer 1965; Oberst 1967).

Curves I and II in figure 8 refer to *VC* copolymers, the viscoelastic values of which are represented in figure 4. In system I the damping foils have been bonded to sheet aluminium. This system has been optimized for aircraft applications, for instance as sandwich panels in the outer walls of jets. In this case, optimum damping efficiency is demanded in a temperature range which extends from approximately −20 to +60 °C. The three-layer sandwich panel II has been optimized for applications in machinery, operating at elevated temperatures. The temperature bandwidth of this system covers the range from 0 °C to approximately 100 °C. Systems III and IV relate to the self-adhesive vinyl acetate copolymer (figure 6).

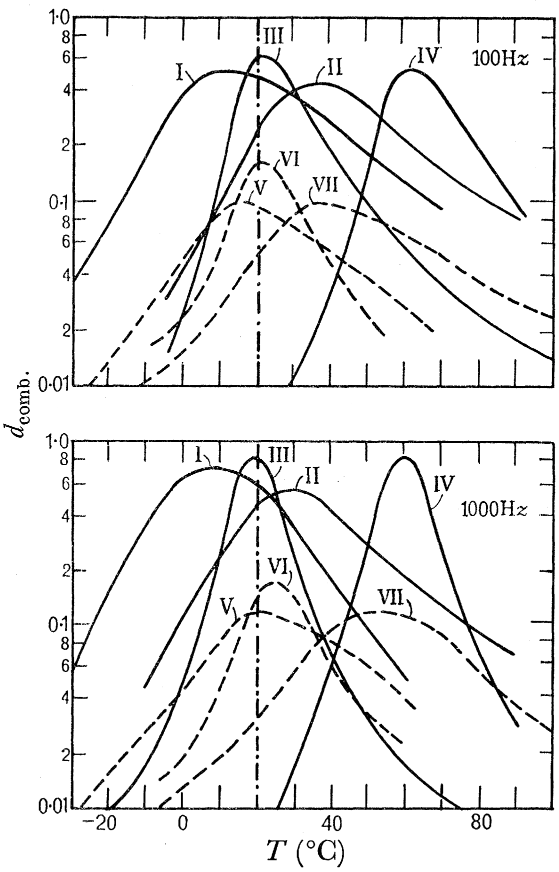

FIGURE 8. Loss factor $d_{comb.}$ of composite systems of metal plates and viscoelastic damping layers. ——, Three-layer sandwich panels; - - - - -, two-layer systems of sheet steel with sprayed-on damping layers, mass ratio of layer to steel = 20 %. Curves I and II, damping foils, bonded to the steel panels, vinyl chloride copolymers of figure 4; curves I, 0·6 mm sheet aluminium, 1·7 mm intermediate layer; curves II, 1 mm sheet steel, 4 mm intermediate layer. Curves III and IV, intermediate layers self-adhesive modified vinyl acetate copolymers, thickness 0·3 mm, 0·5 mm sheet steel. Curves V to VII, damping layers polymers, filled with Vermiculite; curves V, modified vinyl acetate copolymer of figure 3; curves VI, homopoylmer (for comparison); curves VII, modified mixture of polymers.

Curves III are contained in figure 7. The corresponding system has been optimized for the ambient room temperature and is suitable for applications for instance in vehicles. Curves IV relate to a similar self-adhesive viscoelastic material of a higher degree of hardness and are intended for technical applications at elevated temperatures. The optimum range can be shifted between the temperature ranges of curves III and IV by mixing both materials in the melt in a suitable weight ratio. The dashed curves in figure 8 refer to two-layer systems with a mass ratio of the damping layer and the sheet steel of 20 %. Curves V correspond to those represented in figure 3. Curves VII refer to a modified mixture of polymers, filled with Vermiculite. This material has been developed for technical applications at elevated temperatures, too. For comparison curves VI represent a homopolymer filled with Vermiculite, the material being a typical narrow-band damping material. The maxima of $d_{comb.}$ have relatively high values, but the temperature bandwidth is small.

Conclusions

According to the preceding sections the physical laws which govern the viscoelastic properties of polymers and the optimum vibration damping materials made of them, are now well known. This is also true for the damped multilayer structures which are manufactured with those viscoelastic materials. Furthermore, the techniques to apply these results to the development and the manufacture of the various optimized multilayer structures have been worked out to a large extent.

Damping layers on one side are mostly applied to finished metal structures, such as vehicles or machines. This technique has a long tradition and is applied frequently (Oberst *et al.* 1961). Sandwich structures represent a more recent development. Panels of this type will mostly be applied as structural elements in new constructions (Schommer 1966). Already in the present state fairly wide experience has been won with regard to the workability of these panels. Working techniques like riveting, spot welding, forming, bending and deep-drawing are possible (Koch 1966). These sandwich panels have already been used in various technical applications, for instance in vehicles, especially in ships, in industrial environments, e.g. in transformers, conveyors, compressors and the like, in building-techniques, e.g. in air-conditioning lines and in doors of garages (Schommer 1966).

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